Finite Element Analysis of Radius and Ulna

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Outline

• Review bone biomechanics
  ▫ bone behavior during fracture
• Stress analysis concepts
  ▫ properties of bone for models
• Fracture simulations
• Analysis results
• Conclusion
Why Bones Break

- Key relationship: force applied to a structure and displacement

$S \rightarrow$ stiffness

$F_u \rightarrow$ ultimate force

$U \rightarrow$ work to failure

$d_u \rightarrow$ ultimate displacement
Why Bones Break

• Re-express force and displacement as stress and strain

\[ \sigma = \frac{F}{A} = \frac{N}{m^2} = Pa \]

  ▫ 1 Pa ≈ 1 apple on square meter tabletop

• Strain \( \varepsilon = \frac{\Delta l}{L} \)

  ▫ ratio of lengths so unitless
Why Bones Break

- Similar to force-displacement curve
- $E = $ intrinsic stiffness
- Strength is intrinsic property of bone
- Force required to break varies with bone size
- Post yield = permanent
Analysis Concepts

- Young’s Modulus
- Shear Modulus
- Poisson’s Ratio
- No density

\[ E = \frac{F_n/A}{\varepsilon_l} \]

\( E \) = Young's modulus
\( F_n \) = force normal to faces
\( A \) = area
\( \varepsilon_l \) = linear strain
Analysis Concepts

- Young’s Modulus
- Shear Modulus
- Poisson’s Ratio
- No density

\[ S = \frac{F_p}{A} \frac{\varepsilon_s}{\varepsilon_s} \]

- \( S = \) shear modulus
- \( F_p = \) force parallel to faces
- \( A = \) area
- \( \varepsilon_s = \) shear strain
Analysis Concepts

- Young’s Modulus
- Shear Modulus
- Poisson’s Ratio
- No density

\[ \nu = -\frac{\varepsilon_t}{\varepsilon_l} \]

\( \nu = \) Poisson's ratio
\( \varepsilon_t = \) transverse strain
\( \varepsilon_l = \) longitudinal or axial strain
Analysis Concepts

- Ultimate stress increases from transverse to longitudinal force
- Also dependent on tensile or compressive force
Analysis Concepts

- Bone property values

<table>
<thead>
<tr>
<th></th>
<th>Longitudinal</th>
<th>Transverse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's Modulus</td>
<td>17 GPa</td>
<td>11.5 GPa</td>
</tr>
<tr>
<td>Shear Modulus</td>
<td>3.6 GPa</td>
<td>3.3 GPa</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>0.6</td>
<td>0.3</td>
</tr>
</tbody>
</table>

- Ultimate stresses:
  - Longitudinal compression: 200 MPa
  - Longitudinal tension: 140 MPa
  - Transverse compression: 130 MPa
Radius Catching Fall

• Hold proximal end fixed
• Forces applied to underside of distal end
• Expected stresses:
  ▫ Compression on top near constraints
  ▫ Tension on opposite side
Radius Catching Fall

Stress ZZ
- 8.861E+07
- 6.962E+07
- 5.042E+07
- 3.122E+07
- 1.202E+07
- 7.183E+06
- 2.638E+07
- 4.558E+07
- 6.478E+07
- 8.390E+07
- 1.032E+08
Radius Catching Fall

- Von Mises Stress: combines linear and shear stresses from all directions into one value
- Max compression: 180 MPa – 200 MPa
- Max tension: 140 MPa – 160 MPa
- Fracture from tension most likely
- Applied force of 1160 N
Ulna Catching Fall

- Max compression and tension: 140 MPa – 156 MPa
- Tension only cause of fracture
- Applied force of 1610 N
Radius Middle Load

- Proximal and distal ends fixed
- Load applied at midpoint of bone
- Expected stresses:
  - Compression/tension near constraints
  - Transverse compression at midpoint
Radius Middle Load

Stress ZZ
- 8.267E+07
- 6.563E+07
- 4.93E+07
- 3.217E+07
- 1.534E+07
- 1.493E+06
- 1.832E+07
- 3.516E+07
- 5.199E+07
- 6.882E+07
- 8.565E+07
Radius Middle Load

- Max compression: 130 MPa – 145 MPa
- Max tension: 72 MPa – 87 MPa
- Fracture near center due to compression
- Applied force of 4720 N
Ulna Middle Load

- Max compression: 125 MPa – 140 MPa
- Max tension: 140 MPa – 156 MPa
- Fracture near midpoint from compression
- Fracture from tension at wrist
- Applied force of 7450 N
Radius
Compress Ends

- Proximal end fixed
- Rest of bone limited to motion along z-axis
- Expected stresses:
  - compression throughout bone
Radius Compress Ends

Stress ZZ
- 2.509E+07
- 3.1E+07
- 8.708E+07
- 1.432E+08
- 1.993E+08
- 2.554E+08
- 3.114E+08
- 3.675E+08
- 4.236E+08
- 4.797E+08
- 5.358E+08
Radius Compress Ends

- Max compression: 200 MPa – 223 MPa
- No tension
- Fracture near center due to longitudinal compression
- Applied force of 34,200 N
Ulna Compress Ends

- Max compression: 200 MPa – 223 MPa
- No tension
- Fracture more toward distal end than for radius
- Applied force of 40,200 N
Radius Twist

- Proximal end fixed as before
- “Force Couple” used to simulate twist
- Expected stresses
  - Torsion around z-axis
Radius Twist

Stress YZ

-1.598E+07
1.20E+07
9.622E+06
6.446E+06
3.27E+06
9.336E+04
-3.083E+06
-6.259E+06
-9.436E+06
-1.261E+07
-1.579E+07
Radius Twist

- Ultimate stress = 49 – 68 MPa
- Max torsion: 45 MPa – 68 MPa
- Fracture risk for much of bone
- Applied force of 3020 N
- Torque of 54.66 N·m
Ulna Twist

- Use region with only torsion stress
- Max torsion: 45 MPa – 68 MPa
- Fracture from torsion near middle
- Applied force of 4,400 N
- Torque of 78.59 N·m
## Summary of Forces

<table>
<thead>
<tr>
<th></th>
<th>Radius</th>
<th>Ulna</th>
</tr>
</thead>
<tbody>
<tr>
<td>Catching Fall</td>
<td>1,160 N</td>
<td>1,610 N</td>
</tr>
<tr>
<td>Mid</td>
<td>4,720 N</td>
<td>7,450 N</td>
</tr>
<tr>
<td>Comp</td>
<td>34,200 N</td>
<td>40,200 N</td>
</tr>
<tr>
<td>Stretch</td>
<td>23,895 N</td>
<td>28,100 N</td>
</tr>
<tr>
<td>Twist</td>
<td>54.66 N·m</td>
<td>78.59 N·m</td>
</tr>
</tbody>
</table>
Conclusion

• Was able to find fracture locations in radius and ulna for different simulations
• Resulting forces agreed with accepted models of bone anisotropy
• Ulna required larger forces
• Due to bone size and shape possibly described by Wolff’s Law
  ▫ bone in a healthy person or animal will adapt to the loads under which it is placed
Thank You

• Dr. Voytas
• Dr. George
• Dr. Williams
Sources

http://medical-dictionary.thefreedictionary.com/Wolff's+law
http://www.colorado.edu/MCEN/MCEN4173/chap_01.pdf
http://www.engineeringtoolbox.com/poissons-ratio-d_1224.html
http://www.teambone.com/biomechanics.html
http://silver.neep.wisc.edu/~lakes/BoneAniso.html
http://topex.ucsd.edu/geodynamics/shearer.pdf
Questions?
Radius
Stretch Ends

• Same constraints used as in compression
• Direction of force flipped
• Expected stresses:
  ▫ Tension along entire bone
Radius Stretch Ends

Stress ZZ
3.744E+08
3.352E+08
2.96E+08
2.566E+08
2.176E+08
1.784E+08
1.392E+08
1E+08
6.084E+07
2.166E+07
-1.753E+07
Radius Stretch Ends

- No compression
- Max tension: 140 MPa – 156 MPa
- Fracture due to longitudinal tension
- Similar to position of compression fracture
- Applied force of 23,895 N
Ulna Stretch Ends

- No compression
- Max tension: 140 MPa – 156 MPa
- Fracture again closer to wrist
- Applied force of 28,100 N
Analysis Basics

- Stress \( \sigma = \frac{F}{A} \)
  \[ \frac{N}{m^2} = Pa \]
  \[ \Delta l = \frac{\Delta l}{L} \]

- Strain \( \varepsilon = \frac{\Delta l}{L} \)

\[
\sigma = \sqrt{0.5\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2\right] + \sqrt{+3\left(\tau_{xy}^2 + \tau_{yx}^2 + \tau_{zx}^2\right)}}
\]
Why Bones Break

- Biomechanical status of bone may be poorly described by just one of these properties
- Osteopetrotic patient
  - stiff, but brittle
- Young child
  - poorly mineralized and weak, but ductile
Why Bones Break

- Inverse relationship between Young’s modulus and ultimate strain
- Antler: ↓ mineralization, ↑ strain, ↓ E
Why Bones Break

- For my project, using curve for typical bone strength
- Will be looking for ultimate strength value given by program
Current Model

Displacement Magnitude

- 0.00019
- 0.000171
- 0.000152
- 0.000133
- 0.000114
- 9.501E-05
- 7.6E-05
- 5.7E-05
- 3.8E-05
- 1.9E-05
- 0
Current Model

Stress ZZ

- 6.471E+04
- 5.368E+04
- 4.266E+04
- 3.163E+04
- 2.061E+04
- 9586
- -1458
- -1.246E+04
- -2.349E+04
- -3.451E+04
- -4.553E+04
Future Work

• Want to find stresses necessary to cause fractures of arm bones
• Current issues:
  ▫ Thickness of the model
  ▫ Obtaining sensible units
  ▫ Creating realistic fracture scenarios
Future Work

- Want to consider the anisotropic nature of bone
Conclusion

- Bone fractures depend on multiple factors
- Want to find fracture point by examining maximum stress
- Next step is to get meaningful units and realistic fracture models
- Then want to consider anisotropic behavior
Works Cited

http://www.melioguide.com/media/121650/turbo\_bone\_strength.pdf

http://en.wikipedia.org/wiki/Fracture

http://www.brown.edu/Departments/EEB/EML/background/bone.htm
FEA Review

- FEA is a numerical method that provides solutions to difficult problems
  - stress analysis, fluid flow, heat transfer, etc.
- Math behind analysis relies heavily on the use of matrices
- LISA and other programs allow complex computations to be carried out quickly
- Want to use LISA to perform static analysis on bone fractures
FEA Review

- Beam Example
Why Bones Break

1. Ultimate tensile strength
2. Yield strength
3. Proportional limit stress
4. Fracture
5. Offset strain (typically 0.2%)